RODOLFO GUIDOBONO — CURSO TEORÍA CUÁNTICA DE CAMPOS — JAVIER GARCÍA - EJERCICIO CAP. 46 — 11/11/2020

Calcular $\hat{G}(k, x') = (\gamma^\mu k_\mu - mc)^{-1}$

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \quad \gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} \quad \gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Inversa de una matriz $\quad M^{-1} = \dfrac{(\mathrm{Adj}(M))^T}{\det M}$

$$M = \gamma^0 k_0 + \gamma^1 k_1 + \gamma^2 k_2 + \gamma^3 k_3 - mc . \mathbb{I}$$

$$M = \begin{pmatrix} k_0 - mc & 0 & k_3 & k_1 - ik_2 \\ 0 & k_0 - mc & k_1 + ik_2 & -k_3 \\ -k_3 & -k_1 + ik_2 & -k_0 - mc & 0 \\ -k_1 - ik_2 & k_3 & 0 & -k_0 - mc \end{pmatrix}$$

ELEMENTOS DE LA MATRIZ ADJUNTA $\quad \mathrm{Adj}(M)_{ij} = (-1)^{i+j} |M_{ij}|$

$$|M_{1,1}| = \begin{vmatrix} k_0 - mc & k_1 + ik_2 & -k_3 \\ -k_1 + ik_2 & -k_0 - mc & 0 \\ k_3 & 0 & -k_0 - mc \end{vmatrix} = (k_0 - mc)(k_0 + mc)^2 - (k_1 + ik_2)\cdot(-k_1 + ik_2) \cdot(-k_0 - mc) - k_3(0 - k_3(-k_0 - mc)) =$$
$$= (k_0 + mc)\left(k_0^2 - (mc)^2 - (k_1^2 + k_2^2 + k_3^2)\right)$$

$$|M_{1,2}| = \begin{vmatrix} 0 & k_1 + ik_2 & -k_3 \\ -k_3 & -k_0 - mc & 0 \\ -k_1 - ik_2 & 0 & -k_0 - mc \end{vmatrix} = -(k_1 + ik_2)(-k_3)(-k_0 - mc) - k_3(0 + (-k_0 - mc)(-k_1 - ik_2)) =$$
$$= 0$$

$$|M_{1,3}| = \begin{vmatrix} 0 & k_0 - mc & -k_3 \\ -k_3 & -k_1 + ik_2 & 0 \\ -k_1 - ik_2 & k_3 & -k_0 - mc \end{vmatrix} = -(k_0 - mc)(-k_3)(-k_0 - mc) - k_3(-k_3^2 - (-(-k_1 + ik_2)(-k_1 - ik_2))$$
$$= -k_3\left(k_0^2 - (mc)^2 - (k_1^2 + k_2^2 + k_3^2)\right)$$

$$|M_{1,4}| = \begin{vmatrix} 0 & k_0 - mc & k_1 + ik_2 \\ -k_3 & -k_1 + ik_2 & -k_0 - mc \\ -k_1 - ik_2 & k_3 & 0 \end{vmatrix} = -(k_0 - mc)(0 - (-k_0 - mc)(-k_1 - ik_2)) + (k_1 + ik_2)(-k_3^2 - (-k_1 + ik_2)(-k_1 - ik_2))$$
$$= -(k_1 + ik_2)\left(k_0^2 - (mc)^2 - (k_1^2 + k_2^2 + k_3^2)\right)$$

$$|M_{2,1}| = \begin{vmatrix} 0 & k_3 & k_1 - ik_2 \\ -k_1 + ik_2 & -k_0 - mc & 0 \\ k_3 & 0 & -k_0 - mc \end{vmatrix} = -k_3(-k_1 + ik_2)(-k_0 - mc) + (k_1 - ik_2)(0 - k_3(-k_0 - mc))$$
$$= 0$$

$$|M_{2,2}| = \begin{vmatrix} k_0-mc & k_3 & k_1-ik_2 \\ -k_3 & -k_0-mc & 0 \\ -k_1-ik_2 & 0 & -k_0-mc \end{vmatrix} = (k_0-mc)(k_0+mc)^2 - k_3(-k_3)(-k_0-mc) + (k_1+ik_2)\left(0-(-k_0-mc)(-k_1-ik_2)\right) = (k_0+mc)\left(k_0^2-(mc)^2-k_3^2-k_1^2-k_2^2\right)$$

$$|M_{2,3}| = \begin{vmatrix} k_0-mc & 0 & k_1-ik_2 \\ -k_3 & -k_1+ik_2 & 0 \\ -k_1-ik_2 & k_3 & -k_0-mc \end{vmatrix} = (k_0-mc)(-k_1+ik_2)(-k_0-mc) + (k_1+ik_2)\left(-k_3^2-(-k_1+ik_2)(-k_1-ik_2)\right) = (k_1-ik_2)\left(k_0^2-(mc)^2-(k_1^2+k_2^2+k_3^2)\right)$$

$$|M_{2,4}| = \begin{vmatrix} k_0-mc & 0 & k_3 \\ -k_3 & -k_1+ik_2 & -k_0-mc \\ -k_1-ik_2 & k_3 & 0 \end{vmatrix} = (k_0-mc)\left(0-(-k_0-mc)k_3\right) + k_3\left(-k_3^2-(-k_1+ik_2)\times(-k_1-ik_2)\right) = k_3\left(k_0^2-(mc)^2-(k_1^2+k_2^2+k_3^2)\right)$$

$$|M_{3,1}| = \begin{vmatrix} 0 & k_3 & k_1-ik_2 \\ k_0-mc & k_1+ik_2 & -k_3 \\ k_3 & 0 & -k_0-mc \end{vmatrix} = -k_3\left((k_0-mc)(-k_0-mc)+k_3^2\right) + \left(0-(k_1+ik_2)k_3\right)\times(k_1-ik_2) = k_3\left(k_0^2-(mc)^2-(k_1^2+k_2^2+k_3^2)\right)$$

$$|M_{3,2}| = \begin{vmatrix} k_0-mc & k_3 & k_1-ik_2 \\ 0 & k_1+ik_2 & -k_3 \\ -k_1-ik_2 & 0 & -k_0-mc \end{vmatrix} = (k_0-mc)(k_1+ik_2)(-k_0-mc) - k_3\left(0-k_3(k_1+ik_2)\right) + (k_1-ik_2)\left(0-(k_1+ik_2)(-k_1-ik_2)\right) = -(k_1+ik_2)\left(k_0^2-(mc)^2-(k_1^2+k_2^2+k_3^2)\right)$$

$$|M_{3,3}| = \begin{vmatrix} k_0-mc & 0 & k_1-ik_2 \\ 0 & k_0-mc & -k_3 \\ -k_1-ik_2 & k_3 & -k_0-mc \end{vmatrix} = (k_0-mc)\left((k_0-mc)(-k_0-mc)+k_3^2\right) + (k_1-ik_2)\left(0-(k_0-mc)(-k_1-ik_2)\right) = -(k_0-mc)\left(k_0^2-(mc)^2-(k_1^2+k_2^2+k_3^2)\right)$$

$$|M_{3,4}| = \begin{vmatrix} k_0-mc & 0 & k_3 \\ 0 & k_0-mc & k_1+ik_2 \\ -k_1-ik_2 & k_3 & 0 \end{vmatrix} = (k_0-mc)\left(0-k_3(k_1+ik_2)\right) + k_3\left(0-(k_0-mc)(-k_1-ik_2)\right) = (k_0-mc)\left(-k_3(k_1+ik_2)+k_3(k_1+k_2 i)\right) = 0$$

$$|M_{4,1}| = \begin{vmatrix} 0 & k_3 & k_1-ik_2 \\ k_0-mc & k_1+ik_2 & -k_3 \\ -k_1+ik_2 & -k_0+mc & 0 \end{vmatrix} = -k_3\left(0-(-k_3)(-k_1+ik_2)\right) + (k_1-ik_2)\left((k_0-mc)\times(-k_0-mc)-(k_1+ik_2)(-k_1+ik_2)\right) = -(k_1-ik_2)\left(k_0^2-(mc)^2-(k_1^2+k_2^2+k_3^2)\right)$$

$$|M_{4,2}| = \begin{vmatrix} k_0-mc & k_3 & k_1-ik_2 \\ 0 & k_1+ik_2 & -k_3 \\ -k_3 & -k_0-mc & 0 \end{vmatrix} = (k_0-mc)\left(0-(-k_3)(-k_0-mc)\right) - k_3(0-k_3^2) + (k_1-ik_2)\left(0+k_3(k_1+ik_2)\right) = -k_3\left(k_0^2-(mc)^2-(k_1^2+k_2^2+k_3^2)\right)$$

$$|M_{4,3}| = \begin{vmatrix} k_0-mc & 0 & k_1-ik_2 \\ 0 & k_0-mc & -k_3 \\ -k_3 & -k_1+ik_2 & 0 \end{vmatrix} = (k_0-mc)\left(0-(-k_3)(-k_1+ik_2)\right) + (k_1-ik_2)\left(0-(-k_3)(k_0-mc)\right) = 0$$

$$|M_{4,4}| = \begin{vmatrix} k_0 - mc & 0 & k_3 \\ 0 & k_0 - mc & k_1 + ik_2 \\ -k_1 & -k_1 + ik_2 & -k_0 - mc \end{vmatrix} = (k_0 - mc)\big((k_0 - mc)(-k_0 - mc) - (k_1 + ik_2)(-k_1 + ik_2)\big) +$$

$$+ k_3\big(0 - (k_0 - mc)(-k_3)\big)$$

$$= -(k_0 - mc)\big(k_0^2 - (mc)^2 - (k_1^2 + k_2^2 + k_3^2)\big)$$

$$\det M = (k_0 - mc)\,|M_{1,1}| + k_3\,|M_{1,3}| - (k_1 - ik_2)\,|M_{1,4}|$$

$$= (k_0 - mc)(k_0 + mc)\big(k_0^2 - (mc)^2 - (k_1^2 + k_2^2 + k_3^2)\big) +$$

$$+ k_3(-k_3)\big(k_0^2 - (mc)^2 - (k_1^2 + k_2^2 + k_3^2)\big) -$$

$$- (k_1 - ik_2)(k_1 + ik_2)\big(k_0^2 - (mc)^2 - (k_1^2 + k_2^2 + k_3^2)\big)$$

$$= \big(k_0^2 - (mc)^2 - (k_1^2 + k_2^2 + k_3^2)\big)^2$$

$$k_0^2 - (k_1^2 + k_2^2 + k_3^2) = k \cdot k = k^2$$

$$\det M = \big(k^2 - (mc)^2\big)^2$$

$$\mathrm{Adj}(M) = \big(k^2 - (mc)^2\big) \begin{pmatrix} k_0 + mc & 0 & -k_3 & -(k_1 + ik_2) \\ 0 & k_0 + mc & -(k_1 - ik_2) & k_3 \\ k_3 & k_1 + ik_2 & -(k_0 - mc) & 0 \\ k_1 - ik_2 & -k_3 & 0 & -(k_0 - mc) \end{pmatrix}$$

$$M^{-1} = \frac{1}{k^2 - (mc)^2} \begin{pmatrix} k_0 + mc & 0 & k_3 & k_1 - ik_2 \\ 0 & k_0 + mc & k_1 + ik_2 & -k_3 \\ -k_3 & -(k_1 - ik_2) & -(k_0 - mc) & 0 \\ -(k_1 + ik_2) & k_3 & 0 & -(k_0 - mc) \end{pmatrix}$$

$$M^{-1} = \frac{1}{k^2 - (mc)^2}\left(\gamma^0 k_0 + \gamma^1 k_1 + \gamma^2 k_2 + \gamma^3 k_3 + mc\,\mathbb{I}\right)$$

$$\boxed{\hat{G}(k, x') = \frac{\gamma^\mu k_\mu + mc}{k^2 - (mc)^2}}$$